Thermal Emission Characteristics of a Nonisothermal Dielectric Coating on a Conductor Surface

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A heat-transfer analysis of a nonscattering dielectric coating is presented to show the effect of the temperature distribution on the thermal emission characteristics. The problem is formulated using radiation transfer theory for an absorbing, emitting, gray coating with specular surfaces on a metallic substrate. Heat transfer by simultaneous conduction and radiation is considered within the coating. Convection and incident radiation at the coating surface are also included. Results are presented for the temperature distribution in the coating and the thermal emission characteristics. The results indicate that the emission can vary considerably with the temperature distribution. It is shown that a linear temperature approximation can be used to estimate the emission characteristics for the range of parameters investigated.

Nomenclature

a = variable in Eq. (25), τ_0/μ
b = variable in Eq. (25), $1 - \rho_1 e^{-2\tau_0/\mu}$
c = variable in Eq. (25), $1 + \rho_1 e^{-2\tau_0/\mu}$
E_2 = exponential integral function $E_2(\tau) = \int_0^1 \exp\left(\frac{-\tau}{\mu}\right) d\mu$
h^* = dimensionless heat-transfer coefficient, $h/n^2\sigma T_r^3$
h = convective heat-transfer coefficient at the coating surface
I = intensity of radiation
I_b = intensity of blackbody radiation, $\sigma T^4/\pi$
k = thermal conductivity
L = coating thickness
N = dimensionless conduction-radiation interaction parameter, defined by Eq. (15).
n = index of refraction
q = total heat flux
T = absolute temperature
t = dummy integration variable
y = distance from conductor surface
α = absorptivity, ϵ for isothermal coating
ϵ = emissivity defined by Eq. (20)
$\Delta\Theta$ = dimensionless temperature difference defined by Eq. (24)
Θ = dimensionless temperature, T/T_r
θ = angle between outward surface normal and pencil of radiation within the coating
θ' = angle between outward surface normal and pencil of radiation in the adjoining medium
ι = dimensionless intensity, $I/n^2 I_{br}$
κ = absorption coefficient
μ = $\cos\theta$
μ' = $\cos\theta'$
ξ = dimensionless distance, y/L
ρ_i = reflectivity at the i interface for radiation incident on the interface from within the coating
σ = Stefan-Boltzmann constant
τ = optical depth, defined by Eq. (7)
τ_0 = optical thickness, defined by Eq. (8)
Φ = dimensionless radiative flux, $F/n^2\pi I_{br}$
ψ = dimensionless total heat flux, $q/n^2\sigma T_r^4$

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Subscripts

conv = convection
e = reference for emissivity
g = adjoining fluid
h = hemispherical
i = incident
r = reference
1 = conductor or conductor-coating interface
2 = adjoining medium or adjoining medium-coating interface

Superscripts

$+$ = forward direction, $\mu > 0$
$-$ = backward direction, $\mu < 0$

Introduction

RECENT technological developments have emphasized the need for a better understanding of radiation heat transfer. The radiation characteristics of a dielectric coating on a metallic substrate are of considerable practical importance. High-temperature coatings are used to protect a metal surface from its environment or to provide thermal control. In low-temperature applications, a coating may occur, for example, in the form of a condensed gas deposit on a low-temperature surface. The presence of a coating can greatly alter the thermal emission characteristics of a surface. In some high- and low-temperature applications, radiation heat transfer may be of primary importance so that an accurate knowledge of the emission characteristics of a dielectric coating on a metallic substrate is essential.

An analysis has been performed by Francis and Love[1] to determine the radiation characteristics of an isothermal, nonscattering dielectric coating with specular surfaces. They used radiation transfer theory to predict the monochromatic directional emissivity and absorptivity of the coating. Merriam and Viskanta[2] have carried out a similar analysis to investigate the effect of scattering in a coating. In this analysis both diffuse and specular surfaces were considered.

In many applications the coating will not be isothermal. Heat may be transferred from the coating by radiation exchange with the environment and/or convection at the coating surface. The heat is transferred through the coating by conduction and radiation causing a nonuniform temperature

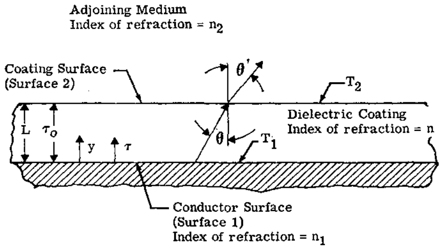

Fig. 1 Physical model and coordinate system.

distribution in the coating. The radiation leaving the surface of the coating depends on the temperature distribution in the coating, since the radiation is emitted from within the coating and from the conductor surface. Thus, in order to predict thermal emission it is necessary to determine the temperature distribution in the coating. Klein[3] has presented an analysis of a nonisothermal ceramic coating which scatters as well as absorbs and emits radiation. He used a "two-flux" approximation for radiation transfer and a linearized energy equation which were previously employed by Hamaker.[4] Klein has shown qualitatively the temperature distribution through such a ceramic coating for various boundary conditions and optical thicknesses. Hobbs et al.[5] have considered the thermal emission from a nonisothermal ceramic material using the "two-flux" approximation and compared their results with experimental measurements. It was found that, for the range of parameters investigated, the energy leaving the ceramic material varied significantly with the temperature gradient within the material for the same surface temperature. For a more complete literature survey, including the effects of scattering, the reader is referred to Dillenius[6] and Bergquam.[7]

Despite the work described here, it appears that no rigorous analysis has been reported for a nonisothermal, nonscattering dielectric coating on a conductor surface. The purpose of this paper is to present such an analysis. The paper includes a formulation of the problem based on radiation transfer theory to predict the temperature distribution in the coating. An expression is derived for the emission based on the temperature distribution. Also, an expression is derived for the emission assuming a linear temperature variation in the coating. Results are presented to illustrate the effect of the nonuniform temperature distribution on the emission and to show the range of parameters where the assumed linear temperature variation is valid.

In the paper, the word "emissivity" will be used to denote the emission. Generally thermal emission from a solid body is characterized by an emissivity based on the surface temperature. Since thermal emission is a volume rather than a surface phenomenon, the use of the surface temperature as a basis for the emissivity is valid only for isothermal opaque bodies or for very strongly absorbing nonisothermal bodies. Emissivity will be employed here as a matter of convenience to denote the emission, although the coating is neither isothermal nor highly absorbing. The reference temperature upon which the emissivity is based will either be the conductor temperature or the coating surface temperature.

Analysis

Physical Model and Assumptions

A schematic diagram of the physical model and coordinate system is shown in Fig. 1. The interfaces of the coating are assumed to be isothermal, to be optically smooth, and to reflect thermal radiation specularly according to the predictions of classical electromagnetic theory. The dielectric coating is considered to be homogeneous and continuous; to absorb and emit, but not scatter thermal radiation; to be in local thermodynamic equilibrium; and to have an index of refraction of n. The thickness of the coating is much greater than a wavelength of radiation. Convective heat transfer may occur at the coating surface and diffuse radiation flux may be incident on the surface from the environment. Heat transfer within the coating is assumed to be one-dimensional and steady.

The problem will be formulated for the gray case where the absorption coefficient for the coating and the indices of refraction for the conductor, coating, and environment are independent of frequency. Also in the analysis the properties are assumed to be independent of temperature. The latter assumption is considered to be reasonable in view of the fact that the temperature drop across the coating is expected to be small.

Basic Equations

The conservation of energy equation for steady, one-dimensional heat transfer within the coating may be written as[8]

$$\frac{dq}{dy} = \frac{d}{dy}\left(-k\frac{dT}{dy} + F\right) = 0 \tag{1}$$

where $-k(dT/dy)$ represents the conductive flux and F the radiative flux. Equation (1) can be integrated once to give

$$q = -k(dT/dy) + F = \text{const} \tag{2}$$

The boundary conditions on Eq. (2) are

$$T(0) = T_1 \tag{3}$$

$$-k(dT/dy)|_{y=L} = h(T_2 - T_g) = q_{\text{conv}} \tag{4}$$

where h is the convective heat-transfer coefficient at the coating surface (surface 2), T_2 is the temperature of surface 2, and T_g is the environment fluid temperature.

An expression for the radiative flux F can be obtained from radiation transfer theory. For a one-dimensional plane layer having axial symmetry the equation of transfer is

$$\mu[dI(y,\mu)/dy] = \kappa[n^2 I_b(y) - I(y,\mu)] \tag{5}$$

The radiative flux is then given by

$$F(y) = 2\pi \int_{-1}^{1} I(y,\mu)\mu d\mu \tag{6}$$

Introducing the optical depth τ defined as

$$\tau = \int_0^y \kappa dy \tag{7}$$

and the optical thickness τ_0 defined as

$$\tau_0 = \int_0^L \kappa dy \tag{8}$$

the dimensionless radiative flux may be written as

$$\Phi(\tau) = \frac{F(\tau)}{\pi n^2 I_{br}} = \frac{F(\tau)}{n^2 \sigma T_r^4} =$$

$$2\left[\int_0^1 \iota^+(0,\mu)e^{-\tau/\mu}\mu d\mu - \int_0^1 \iota^-(0,-\mu)e^{-(\tau_0-\tau)/\mu}\mu d\mu + \int_0^{\tau_0} \Theta^4(t)\,\text{sign}(\tau - t)E_2(|\tau - t|)dt\right] \tag{9}$$

where $\text{sign}(\tau - t) = +1$ for $(\tau - t) > 0$ and $\text{sign}(\tau - t) = -1$ for $(\tau - t) < 0$, and

$$\Theta^4(\tau) = \pi n^2 I_b(\tau)/\pi n^2 I_{br} = T^4(\tau)/T_r^4 \tag{10}$$

The boundary conditions on Eq. (9) are as follows:

$$\iota^+(0,\mu) = [1 - \rho_1(\mu)]\Theta_1^4 + \rho_1(\mu)\iota^-(0,-\mu) \text{ for } \mu > 0 \tag{11}$$

$$\iota^-(\tau_o,\mu) = [1 - \rho_2(\mu)]\iota_i + \rho_2(\mu)\iota^+(0,\mu) \text{ for } \mu < 0 \tag{12}$$

where ι_i is related to the incident diffuse flux by

$$\iota_i = [(F_i/\pi)/n^2\sigma T_r^4](n^2/n_2^2) = [(F_i/\pi)/n_2^2\sigma T_r^4] = \Phi_i/\pi \quad (13)$$

The parameters ρ_1 and ρ_2 in Eqs. (11) and (12) are the directional reflectivities for interfaces 1 and 2, respectively. These reflectivities are functions of μ and are predicted from the Fresnel equations.[8] Equation (9) with the boundary conditions, Eqs. (11–13), define the dimensionless radiative flux.

Introducing dimensionless variables, the conservation of energy equation may be rewritten as

$$(-4N/\tau_0)(d\Theta/d\xi) + \Phi(\xi) = \psi = \text{const} \quad (14)$$

where $\xi = y/L$ and the dimensionless parameter N is defined by

$$N = k\kappa/4n^2\sigma T_r^3 \quad (15)$$

The parameter N indicates the relative magnitude of energy transfer by conduction to that by radiation. The boundary conditions, Eqs. (3) and (4), can be written in dimensionless form as

$$\Theta(0) = \Theta_1 \quad (16)$$

$$(-4N/\tau_0)(d\Theta/d\xi)|_{\xi=1} = h^*(\Theta_2 - \Theta_g) = \psi_{\text{conv}} \quad (17)$$

where h^* is a dimensionless heat-transfer coefficient.

Method of Solution

The temperature distribution in the coating, $\Theta(\xi)$, is given by Eq. (14) with Φ defined by Eq. (9). The expression for Θ is a nonlinear integrodifferential equation. Solutions for $\Theta(\xi)$ were obtained in this analysis by first integrating Eq. (14) once with respect to ξ from 0 to ξ yielding

$$\Theta(\xi) = \Theta_1 + \frac{\tau_0}{4N}\left[\int_0^\xi \Phi(t)dt - \psi\xi\right] \quad (18)$$

The total dimensionless heat flux, ψ, can be related to the boundary conditions by combining Eqs. (14) and (18) evaluated at $\xi = 1$ with Eq. (17). The resulting expression for ψ is

$$\psi = \left[\Phi(\xi) + h^*\left\{\Theta_1 - \Theta_g + \frac{\tau_0}{4N}\int_0^1 \Phi(\xi)d\xi\right\}\right]\left[1 + \frac{h^*\tau_0}{4N}\right] \quad (19)$$

Equation (18), with Φ defined by Eq. (9) and ψ by Eq. (19), was solved numerically using an iterative technique similar to that described by Viskanta and Grosh.[9] The numerical integrations involved in obtaining the solutions were performed using Simpson's rule.[10]

Emissivity

The directional emissivity for the physical model considered can be related to the intensity inside the coating at the surface by

$$\epsilon(\mu') = \{[1 - \rho_2(\mu)]I^+(\tau_0,\mu)\cdot(n_2^2/n^2)\}/n_2^2I_{be} \quad (20)$$

where I_{be} is based on T_e, the reference temperature for the emissivity, and μ' is the cosine of the angle of emission in the adjoining medium. The parameters μ and μ' are related by Snell's law. Equation (20) may be written in terms of dimensionless quantities as

$$\epsilon(\mu') = [1 - \rho_2(\mu)]\iota^+(1,\mu)/\Theta_e^4 \quad (21)$$

The resultant expression for the directional emissivity is

$$\epsilon(\mu') = \frac{(1 - \rho_2)e^{-\tau_0/\mu}}{\Theta_e^4(1 - \rho_1\rho_2e^{-2\tau_0/\mu})}\left[(1 - \rho_1)\Theta_1^4 + \frac{\tau_0}{\mu}\int_0^1 (e^{\xi\tau_0/\mu} + \rho_1e^{-\xi\tau_0/\mu})\Theta^4(\xi)d\xi\right] \quad (22)$$

where ρ_1 and ρ_2 are functions of μ. Once the temperature distribution is known the integral in Eq. (22) can be evaluated numerically and the directional emissivity can be predicted. The hemispherical emissivity, ϵ_h, can be found by numerically integrating $[\epsilon(\mu')\cdot\mu'/\pi]$ over the hemisphere.

Linear Approximation

To obtain results for the temperature distribution and emissivity using the equations derived previously is time-consuming even on a high-speed digital computer. It is thus desirable to find approximate analytical solutions. One such solution is obtained by assuming that the temperature varies linearly within the coating, i.e.,

$$\Theta(\xi) = \Theta_1 + \Delta\Theta\xi \quad (23)$$

where

$$\Delta\Theta = \Theta_2 - \Theta_1 \quad (24)$$

This linear temperature variation would occur if conduction within the coating predominates over radiation, i.e., large values of the parameter N. The results of Viskanta and Grosh[9] indicate that conduction predominates for $N \geq 10$. Substituting Eq. (23) into Eq. (22) and performing the indicated integrations yields

$$\epsilon(\mu') = [(1 - \rho_2)/(1 - \rho_1\rho_2e^{-2a})\Theta_e^4]\{b\Theta_1^4 + 4\Theta_1^3\Delta\Theta[b - c/a + (1 + \rho_1)e^{-a}/a] + 6\Theta_1^2(\Delta\Theta)^2[b(1 + 2/a^2) - 2c/a - 2(1 - \rho_1)e^{-a}/a^2] + 4\Theta_1(\Delta\Theta)^3[b(1 + 6/a^2) - 3c(1/a + 2/a^3) + 6(1 + \rho_1)e^{-a}/a^3] + (\Delta\Theta)^4 \times [b(1 + 12/a^2 + 24/a^4) - 4c(1/a + 6/a^3) - 24(1 - \rho_1)e^{-a}/a^4]\} \quad (25)$$

where

$$a = \tau_0/\mu,\ b = 1 - \rho_1e^{-2\tau_0/\mu},\ c = 1 + \rho_1e^{-2\tau_0/\mu}$$

Although Eq. (25) is complex algebraically, it does give an explicit relationship for the directional emissivity as a function of $\Delta\Theta$ which does not require numerical integration. In order to solve Eq. (25) the temperature difference across the coating must be known. In some applications $\Delta\Theta$ can be measured so that Eq. (25) may be used directly. If $\Delta\Theta$ is unknown, it could be estimated by equating the energy transferred through the coating to the net heat transfer at the coating surface, i.e.,

$$\psi = \frac{-4N}{\tau_0}\frac{d\Theta}{d\xi} + \Phi = h^*(\Theta_2 - \Theta_g) + \epsilon_h\Theta_e^4\left(\frac{n_2^2}{n^2}\right) - \alpha_h\Phi_i\left(\frac{n_2^2}{n^2}\right) \quad (26)$$

where α_h is the hemispherical absorptivity of the coating which is equal to ϵ_h for $\Delta\Theta = 0$. Assuming conduction within the coating predominates $[|(4N/\tau_0)d\Theta/d\xi| \gg \Phi]$, Eq. (26) can be reduced to

$$\Delta\Theta = \{-h^*[\Theta_1 - \Theta_g] - \Theta_e^4(\epsilon_h - \alpha_h\Phi_i) \times (n_2^2/n^2)\}/(h^* + 4N/\tau_0) \quad (27)$$

The parameters ϵ_h and α_h in Eq. (27) must be determined by numerically integrating Eq. (25) times μ'/π over the hemi-

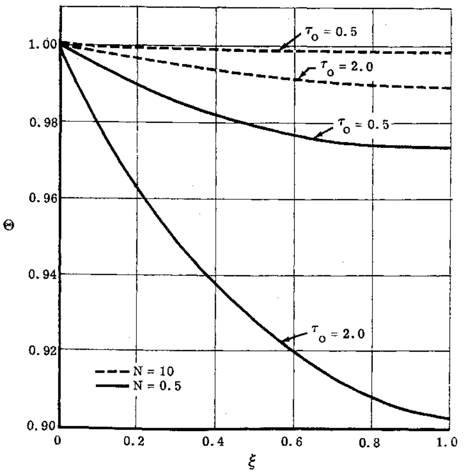

Fig. 2 Effect of optical thickness and parameter N on the temperature distribution in the coating for $n_1 = 1.31 + i10.7$, $n = 1.5$, $n_2 = 1.0$, $h^* = 0$, and $\Phi_i = 0$.

sphere. Equation (27) gives an implicit expression for $\Delta\Theta$ since ϵ_h is a function of $\Delta\Theta$. The temperature difference can be found by iterating on $\Delta\Theta$, or by combining Eqs. (25) and (27), performing the numerical integrations, and solving for the roots of the resulting polynomial expression.

Results

Independent Parameters

Before presenting the results, the independent parameters and the additional assumptions made in obtaining the numerical solutions will be discussed. It is clear from Eq. (18) that the temperature distribution in the coating is a function of the following parameters: τ_0, N, ρ_1, ρ_2, Θ_1, Θ_g, h^*, and Φ_i. The parameters ρ_1 and ρ_2 are functions of the indices of refraction of the metallic substrate, dielectric and the adjoining medium. Thus $\Theta(\xi)$ is a function of nine independent parameters. The parameters h^* and Θ_g could be combined to give one independent parameter ψ_{conv}, but it is felt that in actual applications, h^* and Θ_g would be more likely known or could be more easily estimated than the convective flux at the coating surface. The directional emissivity $\epsilon(\mu')$, as given by Eq. (22), is a function of Θ_e, ρ_1, ρ_2, and τ_0 in addition to the temperature distribution in the coating. The variation of the emissivity with μ' is then a function of ten independent parameters. Similarly, the hemispherical emissivity is a function of the same ten parameters.

In this analysis, it was considered impractical to vary each of the independent parameters over its range of interest. Instead, only the basic parameters τ_0 and N were varied systematically over ranges of interest, while h^* and Θ_g were changed to give desired temperature differences across the coating. The conductor temperature Θ_1 was assumed to be unity and therefore the various dimensionless parameters are referenced to this temperature. The indices of refraction were held constant and assumed to be $1.31 + i10.7$ for the conductor, 1.5 for the coating, and 1.0 for the adjoining medium. These values correspond to one of the cases considered by Francis and Love,[1] and were chosen for the purpose of comparison. The index of refraction of the conductor corresponds to gold at a wavelength of 2 μ. Different indices of refraction would quantitatively affect the results presented in this paper, but not the general trends.

Temperature Distribution

Typical results for the temperature distribution in the coating are shown in Fig. 2. This figure illustrates the temperature variation through the coating in the absence of convection or incident radiation at the surface of the coating. The temperature gradient at $\xi = 1$ vanishes as required by the boundary condition, Eq. (17). This figure indicates that the temperature drop across the coating increases as the optical thickness τ_0 increases and N decreases. With increasing optical thickness a larger fraction of the radiation leaving the surface is emitted by layers within the coating rather than by the conductor. The temperature drop across the coating then increases as more of the radiation leaving the surface is emitted by the coating, and as a larger fraction of the heat is being transferred in the coating by radiation (larger N). It can be seen from Fig. 2 that the temperature drop across the coating can be almost ten percent of the temperature level for the range of parameters considered. Dillenius[6] and Bergquam[7] have measured a temperature drop of about 40% across a layer of alumina powder, and therefore the results predicted are not unreasonable.

The effect of convection at the coating surface on the temperature distribution in the coating is shown in Figs. 3

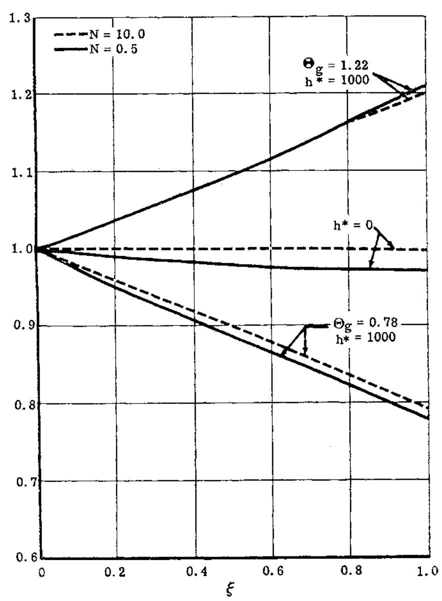

Fig. 3 Effect of convection at the coating surface on the temperature distribution in the coating for $\tau_0 = 0.5$, $n_1 = 1.31 + i10.7$, $n = 1.5$, $n_2 = 1.0$, and $\Phi_i = 0$.

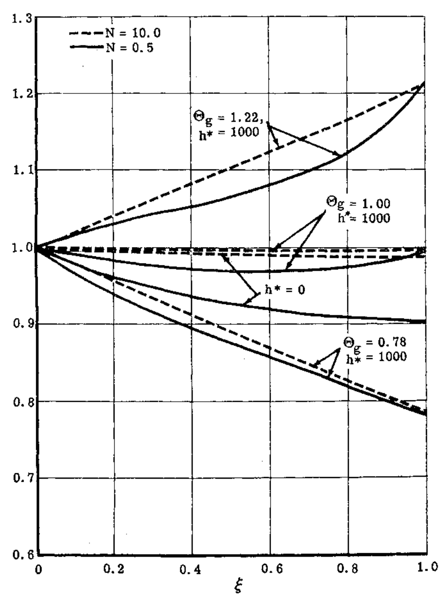

Fig. 4 Effect of convection at the coating surface on the temperature distribution in the coating for $\tau_0 = 2.0$, $n_1 = 1.31 + i10.7$, $n = 1.5$, $n_2 = 1.0$, and $\Phi_i = 0$.

and 4. In these figures, a large h^* was employed so that Θ_g would be nearly equal to Θ_2 and a desired temperature difference across the coating $\Delta\Theta$ could be readily obtained. The range of temperature difference across the coating was selected arbitrarily but was chosen large enough to show the effect of the various parameters. The range of $\Delta\Theta$ shown in Figs. 3 and 4 is the maximum considered in this analysis. Figures 3 and 4 indicate that for $h^* > 0$, the temperature distribution is nearly linear for $N = 10$ as would be expected since conduction is predominate. For $N = 0.5$, the temperature distribution is nonlinear and the deviation from a linear temperature profile increases as τ_0 increases. The results plotted in Fig. 4 for $\Theta_g = 1$ correspond to the case where the surface is heated convectively such that $\Theta_2 \simeq \Theta_1$. The minimum in the temperature distribution is due to a loss of radiant energy from the surface of the coating which has been emitted by layers within the coating.

Figure 5 illustrates the effect of convection on the over-all temperature difference across the coating. It is clear from Eq. (26) that for pure conduction within the coating and negligible radiation heat transfer at the surface

$$d\Theta/d\xi = \Delta\Theta = -\psi_{\mathrm{conv}}\tau_0/4N \tag{28}$$

The deviation of the results in Fig. 5 from the predictions based on Eq. (28) indicates the effect of radiation. The actual values of ψ_{conv} required to obtain a certain $\Delta\Theta$ are not shown directly in Fig. 5. The term $\tau_0/4N$ varies from a minimum of 1/80 for $\tau_0 = 0.5$ and $N = 10$ to a maximum of 1.0 for $\tau_0 = 2$ and $N = 0.5$. The dimensionless convective flux ψ_{conv} required to obtain, for example, a 20% temperature difference across the coating would vary from about 16 for $\tau_0 = 0.5$ and $N = 10$ to something less than one for $\tau_0 = 2$ and $N = 0.5$. It can then be seen that for large τ_0 and small N a 20% temperature difference across the coating could be obtained with $q_{\mathrm{conv}} < n^2\sigma T_1^4$.

The effect of diffuse incident radiation flux on the temperature distribution in the absence of convection is shown in Fig. 6 for $\tau_0 = 2.0$. This figure indicates that for $\Phi_i = 1.0$, the coating is isothermal. This corresponds to the case where the environment is an isothermal enclosure which is at the same temperature as the conductor. Under these conditions, there can be no net heat transfer anywhere within the enclosure, and the coating must be isothermal. Comparison of Fig. 6 with Fig. 4 illustrates that the shape of the temperature distribution is a function of the boundary condition at the surface. This is due to the fact that energy is transferred to the coating near the surface when it is heated convectively; whereas for diffuse incident radiation, energy is absorbed by layers within the coating and by the conductor. As a result, the energy transfer required to raise the surface temperature of the coating to a certain level is less for convection than for diffuse incident radiation.

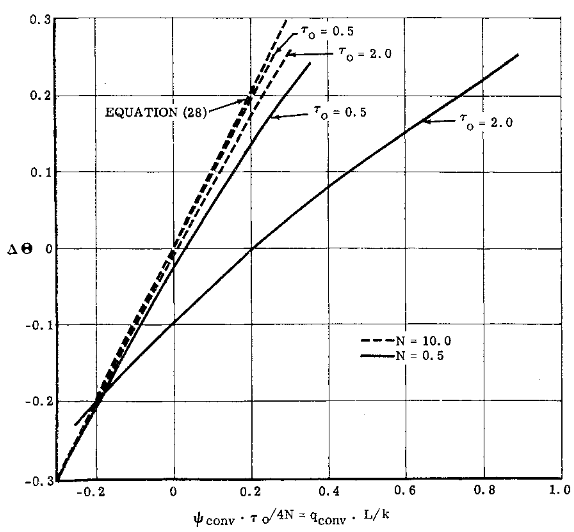

Fig. 5 Dependence of the temperature difference across the coating on the convective heat flux for $n_1 = 1.31 + i10.7$, $n = 1.5$, $n_2 = 1.0$, $\Phi_i = 0$, and $\Theta_1 = 1.0$.

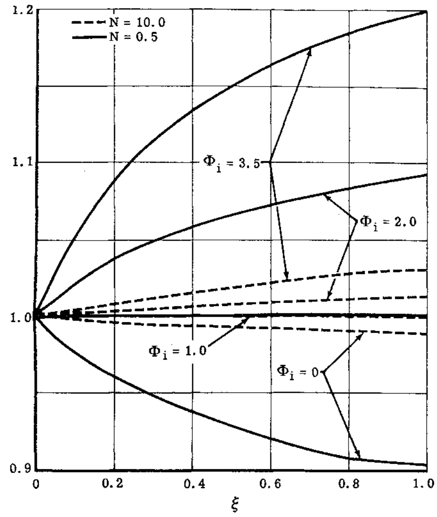

Fig. 6 Effect of diffuse incident radiation flux on the temperature distribution in the coating for $\tau_0 = 2.0$, $n_1 = 1.31 + i10.7$, $n = 1.5$, $n_2 = 1.0$, and $h^* = 0$.

Emissivity

The effect of the temperature distribution in the coating on the emissivity will be illustrated using the temperature difference across the coating $\Delta\Theta$ as an independent parameter. The results to be presented were obtained by varying the convective heat flux at the coating surface to obtain a desired $\Delta\Theta$ and using the resulting temperature distribution to calculate emissivity from Eq. (22).

Figures 7 and 8 show the variation of the directional emissivity with the angle of emission θ'. The reference temperature Θ_e for the emissivity is the conductor temperature Θ_1. The results shown in these figures were obtained using the temperature distributions illustrated in Figs. 3 and 4. The middle curve in each of these polar plots corresponds to the case where there is no convective heat transfer at the surface. It can be seen that for $\tau_0 = 2.0$, the directional emissivity varies with the angle of emission nearly like that of a dielectric. For $\tau_0 = 0.5$, the directional emissivity deviates slightly from that of a dielectric indicating the influence of the conductor. The effect of optical thickness is the same as that observed by Francis and Love.[1] Figures 7 and 8 show that for a given optical thickness, the directional variation of the emissivity is nearly the same for all the values of $\Delta\Theta$ and N considered; only the magnitude is changed. Thus, the effects of the optical thickness and indices of refraction on the directional variation of emissivity of an nonisothermal coating are similar to that of an isothermal coating.

Since $\Delta\Theta$ does not appear to affect the directional variation of the emissivity, the hemispherical emissivity can be used to describe the effects of $\Delta\Theta$ on the thermal emission. Figure 9 illustrates the variation of the hemispherical emissivity based on Θ_1 with $\Delta\Theta$ for different values of τ_0 and the parameter N in the absence of diffuse radiation flux incident on the surface. It can be seen that ϵ_{h1} increases as $\Delta\Theta$ increases. This is due to the fact that as $\Delta\Theta$ increases, the temperature of the dielectric material which is contributing to the emission is increasing relative to the reference temperature Θ_1. As τ_0 increases, the variation of ϵ_{h1} with $\Delta\Theta$ becomes greater since more of the radiation leaving the surface is emitted from the layers closer to the surface. Radiation emitted

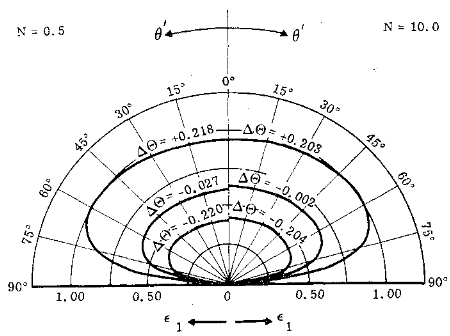

Fig. 7 Effect of temperature difference across the coating on the directional emissivity for $\Theta_e = \Theta_1 = 1.0$, $\tau_0 = 0.5$, $n_1 = 1.31 + i10.7$, $n = 1.5$, $n_2 = 1.0$, and $\Phi_i = 0$.

from layers near the conductor surface is self-absorbed within the coating. Figure 9 also indicates that for a temperature difference across the coating of 20%, ϵ_{h1} will vary from ϵ_{h1} for $\Delta\Theta = 0$ by as much as 40% for $\tau_0 = 0.1$ and 65% for $\tau_0 = 2.0$. This variation with $\Delta\Theta$ is considerable, but is more important for larger τ_0 and smaller N where, as shown by Fig. 5, relatively small values of ψ_{conv} can cause this temperature difference. There is an independent effect of N in this figure for larger values of τ_0 even for $\Delta\Theta = 0$. This effect is due to the temperature deviating from a linear distribution in the coating for small N as shown in Figs. 3 and 4. As τ_0 increases, the deviation from a linear temperature profile increases and also a larger portion of the radiation leaving the surface comes from the coating. This two-fold effect causes ϵ_{h1} to vary more with N at larger values of τ_0 than for the smaller ones. The fact that the effect of N is prevalent for $\Delta\Theta = 0$ indicates that the coating is nonisothermal even though $\Theta_2 = \Theta_1$ (see Fig. 4).

The independent effect of N for $\Delta\Theta = 0$ will be suppressed if diffuse radiation is incident on the surface. Figure 10 illustrates the variation of ϵ_{h1} with $\Delta\Theta$ for $\Phi_i = 1.0$. In this figure, the hemispherical emissivity for $\Delta\Theta = 0$ is not a function of N since the coating is isothermal when $\Delta\Theta = 0$ and $\Phi_i = 1$, as shown in Fig. 6. For $N = 10$ the variation of ϵ_{h1} with $\Delta\Theta$ for $\Phi_i = 1.0$ is almost identical to that for $\Phi_i = 0$. This indicates that when conduction is predominate, diffuse incident radiation does not affect the temperature distribution for a given $\Delta\Theta$.

The results presented above are for an emissivity based on the conductor temperature. This temperature has been used since it is the most likely known or the easiest to measure experimentally. Figure 11 illustrates the variation of the hemispherical emissivity (based on the coating surface temperature) with $\Delta\Theta/\Theta_2$ for different values of τ_0 and N in the absence of incident radiation. The parameter $\Delta\Theta/\Theta_2$ is the

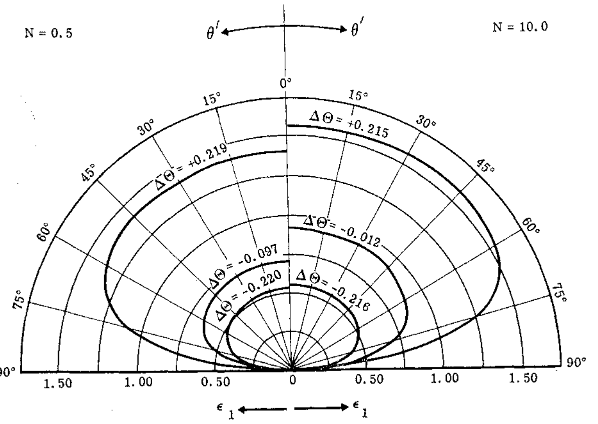

Fig. 8 Effect of temperature difference across the coating on the directional emissivity for $\Theta_e = \Theta_1 = 1.0$, $\tau_0 = 2.0$, $n_1 = 1.31 + i10.7$, $n = 1.5$, $n_2 = 1.0$, and $\Phi_i = 0$.

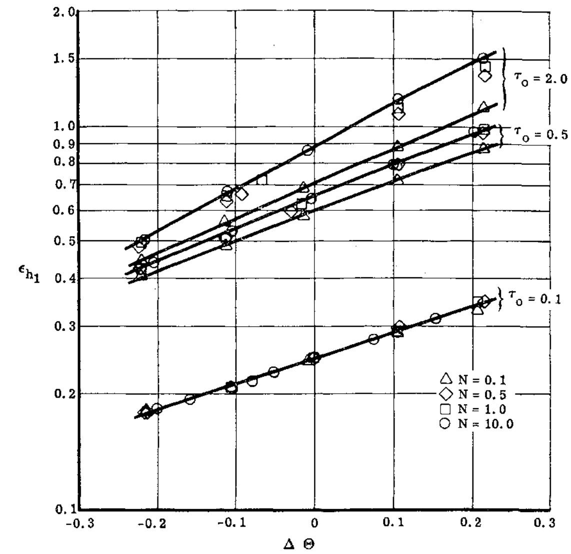

Fig. 9 Variation of the hemispherical emissivity with temperature difference across the coating for $\Theta_e = \Theta_1 = 1.0$, $n_1 = 1.31 + i10.7$, $n = 1.5$, $n_2 = 1.0$, and $\Phi_i = 0$.

independent parameter so that the temperature difference is based on the same temperature as the emissivity. It can be seen that ϵ_{h2} decreases as $\Delta\Theta/\Theta_2$ increases. This is due to the fact that as $\Delta\Theta/\Theta_2$ increases, the temperature within the coating is decreasing relative to Θ_2 and the energy being emitted will decrease relative to black body emission based on Θ_2. The variation of ϵ_{h2} with τ_0 and N is very similar to that shown in Fig. 9 for ϵ_{h1}.

It can be noted that the emissivity in Figs. 7–11 may be greater than unity for certain ranges of the parameters. An emissivity greater than unity is impossible from the definition of emissivity, and this points out the problem when using the concept of emissivity to indicate the emission from a nonisothermal material.

Validity of Linear Approximation

In order to examine the validity of predicting the emissivity using the linear temperature approximation, the hemispherical emissivity was computed by multiplying Eq. (25) by μ'/π and integrating over the hemisphere. It was found that the emissivity obtained using the linear approximation agreed with the emissivity reported in Figs. 9–11 for $N = 10$ within the accuracy of the calculations. This is understandable since for the larger values of $|\Delta\Theta|$ the temperature varies almost linearly in the coating when $N = 10$. When the convective heat transfer is small, the temperature profile is not linear; however, $\Delta\Theta$ is so small that the effect on emissivity for large $N (N \geq 10)$ is negligible. The error in emissivity using the linear approximation for $N < 10$ can be determined from the results shown in Figs. 9–11 by comparing the emissivity for a given N with that for $N = 10$. This error is negligible for $\tau_0 = 0.1$ and is less than ten percent for larger values of τ_0 and $N > 0.5$.

The validity of using Eq. (27) to estimate the temperature drop across the coating has been checked by comparing values of $\Delta\Theta$ obtained from Eq. (27) with results of exact calculations for the temperature distribution. It was found that for large values of h^*, $\Delta\Theta$ predicted from Eq. (27) compares favorably with the exact results since $\Theta_g - \Theta_1$ is nearly equal to $\Delta\Theta$. As $h^* \to 0$, Eq. (27) approximates $\Delta\Theta$ poorly, yielding an error in $\Delta\Theta$ of over 100% for small N. However for $N \geq 0.5$, the error in $\Delta\Theta$ is less than 15% of Θ_1. The effect of an error in $\Delta\Theta$ leads to an error in predicting the emissivity of as much as 25% for $N = 0.5$, 15% for $N = 1$, and 2% for $N = 10$. These errors in emissivity are for $h^* = 0$ and $\tau_0 = 2.0$; they decrease as h^* increases and

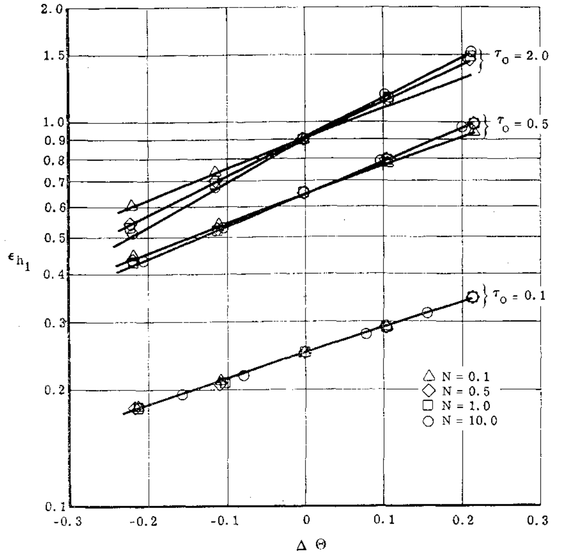

Fig. 10 Variation of the hemispherical emissivity with temperature difference across the coating for $\Theta_e = \Theta_1 = 1.0$, $n_1 = 1.31 + i10.7$, $n = 1.5$ $n_2 = 1.0$, and $\Phi_i = 1.0$.

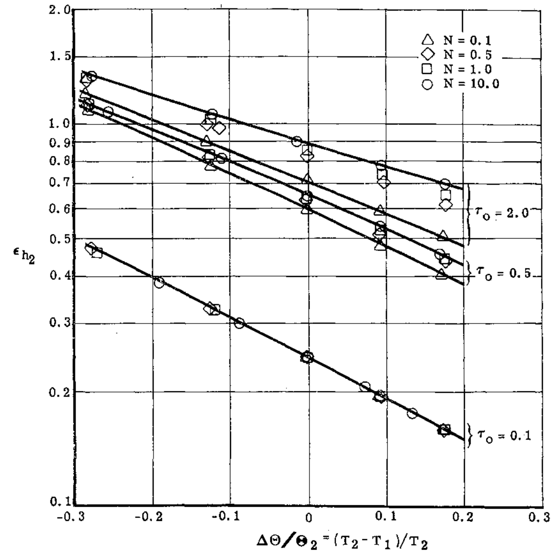

Fig. 11 Variation of the hemispherical emissivity with temperature difference across the coating for $\Theta_e = \Theta_2$, $\Theta_1 = 1.0$, $n_1 = 1.31 + i10.7$, $n = 1.5$, $n_2 = 1.0$, and $\Phi_i = 0$.

τ_0 decreases. Thus Eq. (27) can be used to estimate $\Delta\Theta$ for emissivity predictions for large N and/or small τ_0, or for $h^* \gg 0$.

In this paper, only the gray case has been considered. It is thus assumed that the radiative properties can be replaced by appropriate mean values and considered to be gray. If the radiative properties vary considerably with frequency, the error in using mean values for the properties may or may not be important. For example, the effect of the index of refraction of the substrate varying with frequency could be important for small τ_0 but would be negligible for large τ_0.

Conclusions

The results of this analysis show that the thermal emission characteristics of a coating on a metallic substrate can vary considerably with the temperature distribution in the coating. The temperature distribution is a function of the index of refraction of the coating, conductor, and environment; optical thickness and conduction-radiation interaction parameter; and the boundary conditions at the coating surface. The emissivity is a function of these parameters and as a result varies with the heat transfer within the coating.

It has been shown that for the case considered here where $n/n_2 = 1.5$, the directional dependence of the emissivity on the temperature distribution is small. The hemispherical emissivity then describes the effect of the temperature distribution on the emission characteristics.

The assumed linear temperature approximation can be used to estimate the emission within ten percent for the range of parameters considered. This estimate of the emission improves as the optical thickness decreases and/or the conduction-radiation interaction parameter increases. The linear temperature approximation is based on a known temperature difference across the coating. If the temperature difference is unknown, it can be estimated using Eq. (27) for large N and/or small τ_0, or for $h^* \gg 0$.

Since the emission from the coating is a strong function of the temperature distribution, the use of emissivity is misleading. Thus, caution should be exercised whenever employing the concept of emissivity for a nonisothermal dielectric material.

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